The Superman Punch

Once the fighter launches himself into the air, the trajectory of his shoulder follows a parabola. Suppose the angle between the launching direction and the flat ground is $\theta$, and the launching speed is $v$. The vertical component of the speed is $v \sin \theta$ and the horizontal component of the speed is $v \cos \theta$.

Let $(x, y)$ represent the coordinates of the shoulder and let the initial position of the shoulder be $(0, 0)$. According to Newton’s Laws (neglecting any air resistance),

$$
\begin{cases}
\dot{x} = 0 \\
\dot{y} = -g 
\end{cases}
$$

Where $g$ is the acceleration due to earth’s gravity. Integrating once we have

$$
\begin{cases}
\dot{x} = (v \cos \theta) \\
\dot{y} = -gt + (v \sin \theta)
\end{cases}
$$

(1)

Integrating once more we have

$$
\begin{cases}
x = (v \cos \theta)t \\
y = -0.5gt^2 + (v \sin \theta)t
\end{cases}
$$

(2)

Denote the speed of the shoulder by $u$.

$$
u^2 = (\dot{x})^2 + (\dot{y})^2 = v^2 - (2g\sin \theta)t + g^2t^2
$$

This is a quadratic function with its graph opening upward. Its domain is $[0, \frac{2v\sin \theta}{g}]$. The domain is obtained by solving $y = -0.5gt^2 + (v \sin \theta)t = 0$.

The maximum value of $u^2 = v^2$ occurs at $t = 0$ and $t = \frac{2v\sin \theta}{g}$ (launching and landing).

The minimum value of $u^2 = v^2 \cos^2 \theta$ occurs at $t = \frac{v\sin \theta}{g}$ (at the highest point of the parabola).

Eliminating the parameter $t$ from (2) yields:

$$
y = -\frac{g}{2v^2\cos^2 \theta} x^2 + (\tan \theta)x
$$

Its derivative, i.e., the slope of the tangent line of the trajectory, is
\[
\frac{dy}{dx} = -\frac{g}{v^2 \cos^2 \theta} x + \tan \theta
\]

Which is also the tangent function of the angle \( \alpha \) between the tangent line and the horizontal line,

\[
\tan \alpha = -\frac{g}{v^2 \cos^2 \theta} x + \tan \theta
\]

Or

\[
\alpha = \tan^{-1}\left( -\frac{g}{v^2 \cos^2 \theta} x + \tan \theta \right) = \tan^{-1}\left( -\frac{g}{v \cos \theta} t + \tan \theta \right)
\]

At \( t = \frac{v \sin \theta}{g} \) (when speed is min), \( \alpha = \tan^{-1}(0) = 0 \).

At \( t = \frac{2v \sin \theta}{g} \) (landing, when speed is max), \( \alpha = \tan^{-1}(-\tan \theta) = -\theta \).

At \( t = 0 \) (launching, when speed is max), \( \alpha = \tan^{-1}(\tan \theta) = \theta \).