In 1960, I was an SMSG (School Mathematics Study Group) product, and mathematics enlightened me. Although we’re no longer racing to the moon, maybe it is time to reflect on the direction we are taking basic mathematics. Our goal is similar, to raise the standards of our students. In this case, so that they can compete globally.

The I-Ching (Book of Changes) from which the symbol above comes, recognizes the dynamic balance of opposites, looks at the various means for change, and the acceptance that change is inevitable.
The Tai Chi of Basic Mathematics
(An attempt to find balance)
The Tai Chi of Basic Mathematics
(An attempt to find balance)

Looking at our students.

“I always did poorly in math, so why bother studying. But I need this class for my major, and it’s my third time.”

“This stuff is too easy for me!” I don’t need to study. I’m in the wrong class. Can I leave class early to study for Biology? Biology is a pre-Med requirement.”
Those students with math anxiety may find some success in returning to the fundamentals, but applications and making the connections is still difficult. Over-confident students have realized that they may not moving on.
Looking at the curriculum.

The Tai Chi of Basic Mathematics
(An attempt to find balance)

DRILL and PRACTICE.

APPLICATION PROBLEMS

1. There are 200 passengers on board an airplane. 3/5 of them are men, 1/5 of them are women and the rest are children. How many children are there?

2. There are 350 members in a swimming club. 2/7 of them are new members. 3/10 of the new members are females. How many new female members are there?

3. Sally made 500 cookies. She sold 3/4 of them and gave away 2/5 of the remainder. How many cookies did she give away?

4. Dani made some sticks of satay for a party. 3/5 of them were chicken satay and the rest were beef satay. There were 240 sticks of beef satay. How many sticks of chicken satay were there?

5. After paying $30 for a shirt, David had 3/5 of his money left. How much money did he have at first?

6. After spending 2/5 of his money on a storybook, Mathew had $12 left. How much did he spend on the storybook?
The Tai Chi of Basic Mathematics
(An attempt to find balance)

Looking at a diverse population.

Whether a student’s native language is English or not, many adapt to drill and practice.

DRILL and PRACTICE

APPLICATIONS and READING
For students whose language is NOT English, it is another hurdle. Native born students also have difficulty.
The Tai Chi of Basic Mathematics
(An attempt to find balance)

Looking at PERSISTENCE

MATH ANXIETY
(w/excuse)

ENCOURAGEMENT
(Find a bone for the dog)

HOPEFUL RESULT
(Found a bone for the dog)
The Tai Chi of Basic Mathematics
(An attempt to find balance)

In most cases, there should not be a difference in what we say and what we do. I looked to Albert Einstein for his experience and intellect for what I should be teaching in my classroom. It is still a matter of choice, and I choose to believe the statement below.

“I NEVER SAID HALF THE CRAP PEOPLE SAID I DID”
ALBERT EINSTEIN

Education is not the learning of facts, but the training of the mind to think.
~ Albert Einstein
The Tai Chi of Basic Mathematics
(An attempt to find balance)

In dealing with PROBLEM SOLVING, one has three areas to address.

Create an environment for learning

Promote discovery and long-term understanding

Support and Encourage Persistence
Creating an Environment for Learning

The Tai Chi of Basic Mathematics
(An attempt to find balance)

Creating an Environment for Learning

Yin Cycle  Yang Cycle

Create an environment for learning

Anxiety  Feelings Continuum  Confidence

Psychological/Emotional
The Tai Chi of Basic Mathematics
(An attempt to find balance)
The Tai Chi of Basic Mathematics
(An attempt to find balance)

Addressing Isolation

In each circle, write something that is important to you, and perhaps, you would like others to know about you.
The Tai Chi of Basic Mathematics
(An attempt to find balance)

Addressing Isolation

In each circle, write something that is important to you, and perhaps, you would like others to know about you.

FAMILY
I have wife, Mary, who raises my two daughters, Barbara and Loretta, and my son, Jimmy.
The Tai Chi of Basic Mathematics
(An attempt to find balance)

Addressing Isolation

In each circle, write something that is important to you, and perhaps, you would like others to know about you.

FAMILY
I have wife, Mary, who raises my two daughters, Barbara and Loretta, and my son, Jimmy.

EDUCATION
My parents never finished high school, and I am the oldest of 7 children, all of which except for one went to college and got a degree.
The Tai Chi of Basic Mathematics
(An attempt to find balance)

Addressing Isolation

In each circle, write something that is important to you, and perhaps, you would like others to know about you.

FAMILY
I have wife, Mary, who raises my two daughters, Barbara and Loretta, and my son, Jimmy.

EDUCATION
My parents never finished high school, and I am the oldest of 7 children, all of which except for one went to college and got a degree.

MY JOB
I love my job and mathematics. I am fortunate to be doing what I have been doing for 40 years.
The Tai Chi of Basic Mathematics
(An attempt to find balance)

Addressing Isolation

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EDUCATION
My parents never finished high school, and I am the oldest of 7 children, all of which except for one went to college and got a degree.

“Man Cave”
I’m remodeling my home so that I can have a quiet place other than the garage.
The Tai Chi of Basic Mathematics
(An attempt to find balance)

Promote Discovery and Long-term Understanding

Yin Cycle  Yang Cycle

Failure  Achievement Continuum  Success

Intellectual/Educational
Model mathematics and let their innate skills develop.

“Everything should be made as simple as possible, but not simpler.”

Albert Einstein

“Challenge the Familiar”
A primary component of the Singapore Math course is the use of visual representations of mathematical quantities and its role in problem solving single and multi-step applications.
What is the initial value of the shaded region?

Evaluating the expression when \( x = 9 \)

\[
\frac{3}{5} (3x - 2).
\]

Solving the equation

\[
\frac{2x+5}{3} - 3 = 6.
\]
The Tai Chi of Basic Mathematics
(An attempt to find balance)
Paul and Cheryl are selling hats. Cheryl purchased two boxes of hats while Paul purchased three times as many. If \( \frac{5}{6} \) of Paul’s purchase is 10 hats, how many hats did each of them purchase?

1. Understand the problem
   Paul and Cheryl are selling hats. Cheryl purchased two boxes of hats while Paul purchased three times as many. If \( \frac{5}{6} \) of Paul’s purchase is 10 hats, how many hats did each of them purchase?

2. Devise a plan

3. Carry out the plan

4. Look back
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Paul and Cheryl are selling hats. Cheryl purchased two boxes of hats while Paul purchased three times as many. If 5/6 of Paul’s purchase is 10 hats, how many hats did each of them purchase?

**2. Devise a plan**

<table>
<thead>
<tr>
<th>Cheryl</th>
<th>2 boxes of hats</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paul</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3 times as many as Cheryl</td>
</tr>
<tr>
<td></td>
<td>5/6 of Paul's purchase total 10 hats</td>
</tr>
</tbody>
</table>

**3. Carry out the plan**

<table>
<thead>
<tr>
<th>Cheryl</th>
<th>2</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paul</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

**4. Look back**

Cheryl bought 4 hats and Paul purchased 12 hats.

5/6 of 12 is 10.
The Tai Chi of Basic Mathematics
(An attempt to find balance)

PROBLEM SOLVING
Models → Solutions

Given the information below, what is the value of the shaded regions.

Problem #1
54
117

Problem #2
47
19

Problem #3
35
77

Problem #4
78
32

Problem #5
94
213

Problem #6
83
37

170

17
Fred could not divide the amount of money in his pocket equally among his 4 kids. His wife gave him an additional $3 after which each of his 4 kids received $8.
Fred could not divide the amount of money in his pocket equally among his 4 kids. His wife gave him an additional $3 after which each of his 4 kids received $8.
Tristan divided a certain amount of money into 3 equal shares. He gave the first share to Betty, the second share to Veronica, and he then divided the remainder into 3 equal shares and gave one share to Archie. He kept the rest totaling $24. How much money did Tristan initially have?
The Tai Chi of Basic Mathematics
(An attempt to find balance)

PROBLEM SOLVING (Words → Models → Solutions)

Tristan divided a certain amount of money into 3 equal shares. He gave the first share to Betty, the second share to Veronica, and he then divided the remainder into 3 equal shares and gave one share to Archie. He kept the rest totaling $24. How much money did Tristan initially have?

<table>
<thead>
<tr>
<th>Tristan</th>
<th>Betty</th>
<th>Veronica</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Archie</td>
</tr>
<tr>
<td>$24</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The Tai Chi of Basic Mathematics
(An attempt to find balance)

PROBLEM SOLVING and Linear Models $\rightarrow$ Expression

Given the model, create the algebraic expression.

Illustration:

What is the Algebraic Expression?

1
The Tai Chi of Basic Mathematics
(An attempt to find balance)

PROBLEM SOLVING and Linear Models → Expression

Given the model, create the algebraic expression.

Illustration:

What is the Algebraic Expression?

\[ 2\left(\frac{x}{3} - 1\right) \]
The Tai Chi of Basic Mathematics
(An attempt to find balance)

Given the model, create the algebraic expression.

Illustration:

What is the Algebraic Expression?

\[ X \]
Given the model, create the algebraic expression.

Illustration:

What is the Algebraic Expression?

\[
\frac{2}{3} (5x + 4)
\]
The Tai Chi of Basic Mathematics
(An attempt to find balance)

Linear Functions and the Order of Operations → Model

Given the algebraic expression, label the order of each operation involve in the process, and then create the model.

\[ 3\left(\frac{x}{2} + 5\right) \]

3 Operations
The Tai Chi of Basic Mathematics
(An attempt to find balance)

Given the algebraic expression, label the order of each operation involved in the process, and then create the model.

Linear Functions and the Order of Operations → Model

Op 1 DIVISION

3 Operations

3(\frac{x}{2} + 5)
The Tai Chi of Basic Mathematics
(An attempt to find balance)

Linear Functions and the Order of Operations → Model

Given the algebraic expression, label the order of each operation involved in the process, and then create the model.

3 Operations

Op 1  DIVISION

Op 2  SUM

3(\frac{x}{2} + 5)

3 Operations
The Tai Chi of Basic Mathematics
(An attempt to find balance)

Linear Functions and the Order of Operations → Model

Given the algebraic expression, label the order of each operation involved in the process, and then create the model.

- **Op 1** DIVISION
  - 3

- **Op 2** SUM
  - $\frac{x}{2}$
  - 5

- **Op 3** TIMES
  - 3

3 Operations
The Tai Chi of Basic Mathematics
(An attempt to find balance)

Linear Functions and the Order of Operations \(\rightarrow\) Model

Given the algebraic expression, label the order of each operation involved in the process, and then create the model.

\[ 3 \left( \frac{x}{4} + 1 \right) + 2 \]

5 Operations
The Tai Chi of Basic Mathematics
(An attempt to find balance)

Linear Functions and the Order of Operations → Model

Given the algebraic expression, label the order of each operation involve in the process, and then create the model.

Op 1  DIVIDE

\[ \frac{3}{4} + 1 + 2 \]

5 Operations
The Tai Chi of Basic Mathematics
(An attempt to find balance)

Linear Functions and the Order of Operations → Model

Given the algebraic expression, label the order of each operation involve in the process, and then create the model.

\[ 3 \left( \frac{x}{4} + 1 \right) + 2 \]

5 Operations
The Tai Chi of Basic Mathematics
(An attempt to find balance)

Linear Functions and the Order of Operations → Model

Given the algebraic expression, label the order of each operation involved in the process, and then create the model.

\[ 3 \left( \frac{x}{4} + 1 \right) + 2 \]

1. DIVIDE
2. SUM
3. TIMES

5 Operations
Linear Functions and the Order of Operations → Model

Given the algebraic expression, label the order of each operation involve in the process, and then create the model.

\[ 3 \left( \frac{x}{4} + 1 \right) + 2 \]

Operations:

1. **DIVIDE**
   - \( \frac{x}{4} \)
2. **SUM**
   - \( +1 \)
3. **TIMES**
   - \( \times \)
4. **SUM**
   - \( +2 \)
5. **SUM**
   - \( + \)

Total operations: 5
The Tai Chi of Basic Mathematics
(An attempt to find balance)

Linear Functions and the Order of Operations → Model

Given the algebraic expression, label the order of each operation involve in the process, and then create the model.

\[ 3 \left( \frac{x}{4} + 1 \right) + 2 \]

5 Operations
The Tai Chi of Basic Mathematics
(An attempt to find balance)

PROBLEM SOLVING: Evaluating an Expression.

Given the model and the corresponding input value, determine the value of the expression.

If the value of the x is 90, determine the value of the expression.
The Tai Chi of Basic Mathematics
(An attempt to find balance)

PROBLEM SOLVING: Evaluating an Expression.

Given the model and the corresponding input value, determine the value the expression.

If the value of the $x$ is 7, determine the value of the expression.

Illustration:

\[
\frac{2}{3} (5x + 4)
\]
Given the model and the corresponding input value, determine the value the expression.

If the value of the \( x \) is 7, determine the value of the expression.

\[
\frac{2}{3} (5x + 4)
\]
The Tai Chi of Basic Mathematics
(An attempt to find balance)

PROBLEM SOLVING Models → Solving an Equation

Given the model and value of the expression, determine the value of its input.

\[
\left( \frac{3}{4}x + 4 \right) = 25
\]

The expression has a value of 25; so what is the value of \(x\)?
Given the model and value of the expression, determine the value of its input.

The expression has a value of 25; so what is the value of x?

\[
\left( \frac{3}{4}x + 4 \right) = 25
\]
The Tai Chi of Basic Mathematics
(An attempt to find balance)

PROBLEM SOLVING Models → Solving an Equation

Given the model and value of the expression, determine the value of its input.

Illustration:

The expression has a value of 8; so what is the value of x?

\[ \frac{2}{3} \left( \frac{5}{9}x - 3 \right) = 8 \]
The Tai Chi of Basic Mathematics
(An attempt to find balance)

PROBLEM SOLVING Models → Solving an Equation

Given the model and value of the expression, determine the value of its input.

Illustration:

The expression has a value of 8; so what is the value of x?

\[
\frac{2}{3} \left( \frac{5}{9}x - 3 \right) = 8
\]
The Tai Chi of Basic Mathematics  
(An attempt to find balance)

PROBLEM SOLVING Models → Solving an Equation

Given the model and value of the expression, determine the value of its input.

Illustration:

The expression has a value of 8; so what is the value of x?

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The Tai Chi of Basic Mathematics
(An attempt to find balance)

PROBLEM SOLVING Models → Solving an Equation

Given the model and value of the expression, determine the value of its input.

Illustration:

\[ \frac{2}{3} \left( \frac{5}{9}x - 3 \right) = 8 \]

The expression has a value of 8; so what is the value of \( x \)?
The Tai Chi of Basic Mathematics
(An attempt to find balance)

PROBLEM SOLVING Models → Solving an Equation

Given the model and value of the expression, determine the value of its input.

Illustration:

The expression has a value of 8; so what is the value of x?

\[
\frac{2}{3} \left( \frac{5}{9}x - 3 \right) = 8
\]
Given the model and value of the expression, determine the value of its input.

1) ADD 2 to get 8 for the YELLOW box.
The Tai Chi of Basic Mathematics
(An attempt to find balance)

PROBLEM SOLVING Models → Solving an Equation via the Algebraic Method

Given the model and value of the expression, determine the value of its input.

2) MULTIPLY by 3 to get 24

\[
\frac{(5x - 1)}{3} - 2 = 6
\]

Describe the steps you will need to do to solve for \(x\).
The Tai Chi of Basic Mathematics
(An attempt to find balance)

PROBLEM SOLVING Models \(\rightarrow\) Solving an Equation via the Algebraic Method

Given the model and value of the expression, determine the value of its input.

3) ADD 1 to get 25

\[
\frac{(5x - 1)}{3} - 2 = 6
\]

Describe the steps you will need to do to solve for \(x\).
The Tai Chi of Basic Mathematics
(An attempt to find balance)

PROBLEM SOLVING Models → Solving an Equation via the Algebraic Method

Given the model and value of the expression, determine the value of its input.

4) DIVIDE by 5 to get 5

\[
\frac{(5x - 1)}{3} - 2 = 6
\]

Describe the steps you will need to do to solve for x.

5  x

\[
\begin{array}{cccccc}
\text{x} & \text{x} & \text{x} & \text{x} & \text{x} \\
1 & 24 \\
\text{8} & \text{8} & \text{8} \\
2 & 6
\end{array}
\]

25
4
The Tai Chi of Basic Mathematics
(An attempt to find balance)

PROBLEM SOLVING Models → Solving an Equation via the Algebraic Method

Given the model and value of the expression, determine the value of its input.

1) Subtract 5 from 13 to get the length of the left rectangle.

2 \( 2(x - 3) + 5 = 13 \)

1 3

1 2

1 3

1 5

1 8

1 3

2 5

2 13
The Tai Chi of Basic Mathematics  
(An attempt to find balance)

PROBLEM SOLVING Models → Solving an Equation via the Algebraic Method

Given the model and value of the expression, determine the value of its input.

\[ 2(x - 3) + 5 = 13 \]

1. \[ 2(x - 3) + 5 = 13 \]
2. Divide by 2
Given the model and value of the expression, determine the value of its input.

\[
\begin{align*}
2(x - 3) + 5 &= 13 \\
2x - 6 + 5 &= 13 \\
2x &= 13 + 6 \\
2x &= 19 \\
\end{align*}
\]

3) Add 3
An Important Goal:

*Problem Solving and Critical Thinking w/ Applications*
Jim took $23.00 with him to go shopping. His sister, Loretta took $13.00 more than he did.

How much did Loretta take shopping?
Judy bought some eggs. She used $\frac{2}{5}$ of the eggs to bake cakes. She had 18 eggs left.

How many eggs did she buy?
Tomas took a certain amount of money from his bank account to go shopping at the mall. He spent \( \frac{2}{5} \) of the money on clothing, and \( \frac{2}{3} \) of the remainder for power tools. What fraction of his original amount was left?

What fraction of his original amount was left?
Tom spent three-quarters of his money on a dictionary. He spent one-half of the remainder on a calculator. The dictionary cost $30 more than the calculator.

How much does the dictionary cost?
There are 20 workers in the library. 55% of them were males. How many fewer females than males worked in the library?

10% more males than females employed
Mary determined that the population of monarch butterflies at a particular site was 12,000. She estimated that next year there would be a 6% increase each year.

What would be the estimated population of monarch butterflies next year?

<table>
<thead>
<tr>
<th>100</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>12,000</td>
<td>?</td>
</tr>
</tbody>
</table>
A shopkeeper had 4 handbags which were of the **same cost price**. He sold 3 of them at **40% more than** the cost price. He sold the fourth handbag at cost price. He received a **total of $260** altogether. Find the cost price of each handbag.
Sally is given $5 more allowance than Megan each week. They each spend $12 per week and save the rest. When Sally has saved $60, Megan saved $20.

Find out Sally’s allowance.
In a class, at the beginning of the semester, the ratio of girls to boys is 5:3. If an additional 4 girls and 12 boys enrolled, there would be the same number of girls as boys in the class.

How many girls were there at the beginning of the semester?
Ali had $130 and his brother had $45. When their mother gave each of them an equal amount of money, Ali had twice as much as his brother.

How much did their mother contribute to each of them?
The Tai Chi of Basic Mathematics
(An attempt to find balance)

Support and Encourage Persistence

Support Persistence

Avoidance Behavioral Continuum Pursuit Social/Motivational

Yin Cycle Yang Cycle
The Tai Chi of Basic Mathematics
(An attempt to find balance)

Okay, ready to begin this problem.

It’s tougher than I thought!

I’ve read it over 25 times. Nothing.

I remember now why I hate math.

Hey, maybe someone in my class study group can help me?

My group mates think that we can get together in the quad to work on it.

Solved it and ready to do the next one.

Okay, ready to begin this problem.

The vicious elliptical path problem solving . . .

P E R S I S T E N C E
Half of a Combo-Review (part 1) is taken home giving many students opportunities to develop work groups. In class, part 2 with similar objects is given. A master form where answers are recorded when the groups get together is turned in for credit.

Everybody works, everybody benefits.