

Finally, A Use For Calculus: Intergalactic Warfare

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THE STORY

It is the year 3010 and the Andromeda Galaxy is threatening to take control of the Milky Way Galaxy. The people of Earth, along with over 50 other Earth-like planets have built a fleet of 3,700 armed space ships. All intelligent life in the Andromeda Galaxy has assembled a total of 1,200 space ships.

The Andromeda Galaxy is more technologically advanced than the Milky Way Galaxy and their space ships are three times as powerful as ours. This fact scares the Milky Way Galactic Navy and they have commissioned the math club at Santa Rosa Junior College to calculate the result before commencing battle.

THE MATH

We begin with a system of differential equations where x is the number of our ships, y is the number of Andromeda's ships and t is measured in months. We begin with 3,700 ships and Andromeda begins with 1,200.

$$\begin{cases} \frac{dx}{dt} = -0.12y \\ \frac{dy}{dt} = -0.04x \end{cases}; x(0) = 3700, y(0) = 1200$$

Some algebraic manipulation and use of a differential operator yields the following:

$$\begin{cases} Dx + 0.12y = 0 \\ Dy + 0.04x = 0 \end{cases}$$

We find the general equation for $y(t)$ using elimination and assuming a solution in the form $y = e^{mx}$.

$$\begin{cases} (-0.04(Dx + 0.12y)) = 0 \\ D(Dy + 0.04x) = 0 \end{cases} \rightarrow \begin{cases} -0.04Dx - 0.0048y = 0 \\ 0.04Dx + D^2y = 0 \end{cases}$$

$$D^2y - 0.0048y = 0$$

$$m^2 - 0.0048 = 0$$

$$m = \pm\sqrt{0.0048}$$

$$y(t) = c_1 e^{\sqrt{0.0048}t} + c_2 e^{-\sqrt{0.0048}t}$$

Recalling $\frac{dx}{dt} = -0.12y$, we use integration to find $x(t)$.

$$x' = -0.12(c_1 e^{\sqrt{0.0048}t} + c_2 e^{-\sqrt{0.0048}t})$$

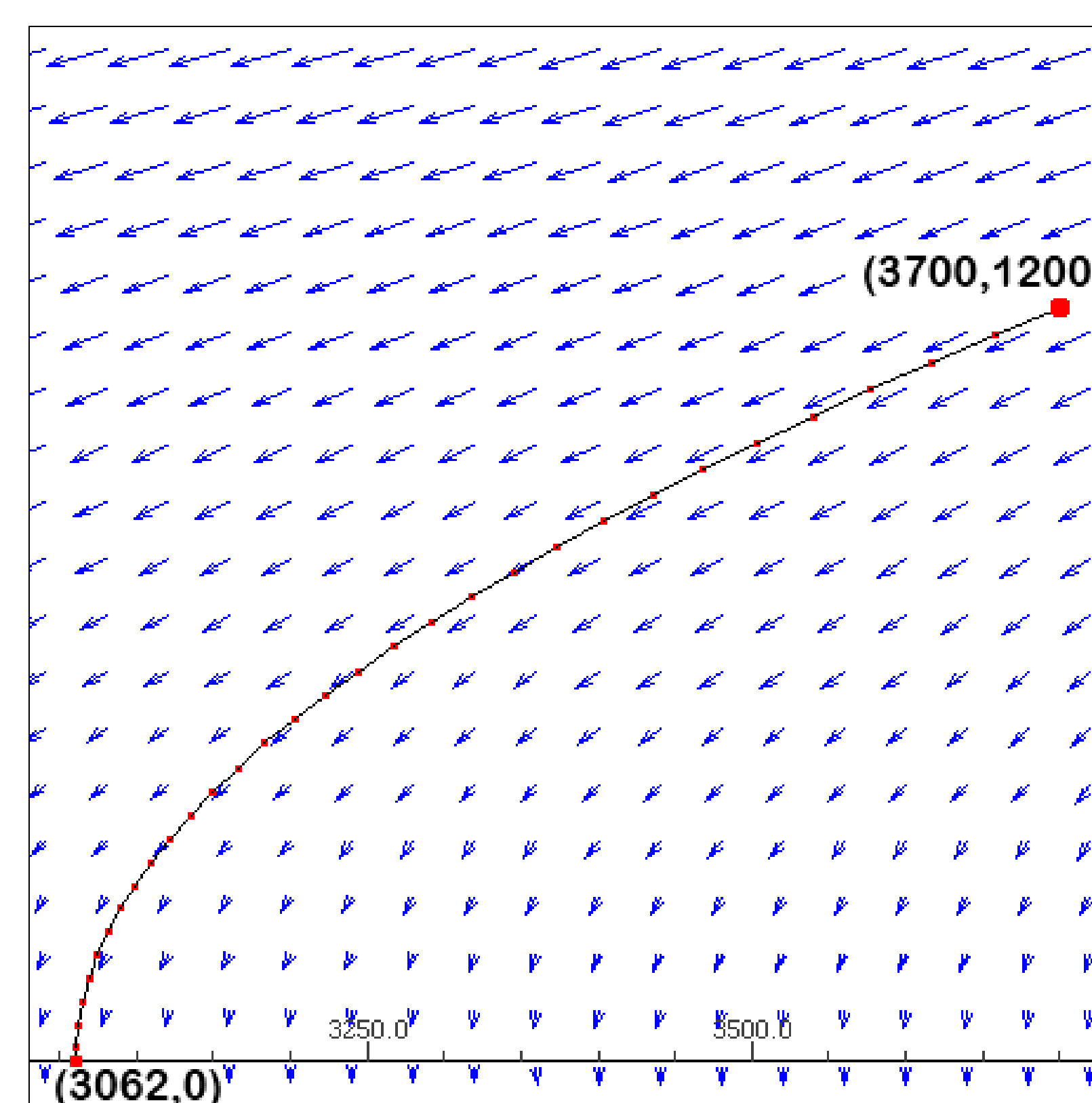
$$\int x' dt = -0.12 \left(c_1 \int e^{\sqrt{0.0048}t} dt + c_2 \int e^{-\sqrt{0.0048}t} dt \right)$$

$$x(t) = -0.12 \left(\frac{c_1}{\sqrt{0.0048}} e^{\sqrt{0.0048}t} - \frac{c_2}{\sqrt{0.0048}} e^{-\sqrt{0.0048}t} \right)$$

$$x(t) = -1.73c_1 e^{\sqrt{0.0048}t} + 1.73c_2 e^{-\sqrt{0.0048}t}$$



Milky Way Ships (x) vs. Andromeda Ships (y)



THE MATH (Cont'd)

Now that we have a general solution for the system, we use our initial conditions of 3,700 ships and 1,200 ships to find the particular solution:

$$\begin{cases} x(t) = -1.73c_1 e^{\sqrt{0.0048}t} + 1.73c_2 e^{-\sqrt{0.0048}t} \\ y(t) = c_1 e^{\sqrt{0.0048}t} + c_2 e^{-\sqrt{0.0048}t} \end{cases}$$

$$\begin{cases} x(t) = -1.73c_1 e^{\sqrt{0.0048}t} + 1.73c_2 e^{-\sqrt{0.0048}t} = 3700 \\ y(t) = c_1 e^{\sqrt{0.0048}t} + c_2 e^{-\sqrt{0.0048}t} = 1200 \end{cases}$$

$$\begin{bmatrix} -1.73 & 1.73 & 3700 \\ 1 & 1 & 1200 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -469.364 \\ 0 & 1 & 1669.36 \end{bmatrix}$$

$$c_1 = -469.364 \quad \& \quad c_2 = 1669.36$$

$$\begin{cases} x(t) = 812 e^{\sqrt{0.0048}t} + 2887.99 e^{-\sqrt{0.0048}t} \\ y(t) = -469.364 e^{\sqrt{0.0048}t} + 1669.36 e^{-\sqrt{0.0048}t} \end{cases}$$

THE RESULTS

To allay the Milky Way Galactic Navy's fears, SRJC Math Club students determine how long the war will last and how many ships will remain. Since Andromeda is a particularly hostile galaxy, the war will cease only when they have zero ships remaining.

$$y(t) = -469.364 e^{\sqrt{0.0048}t} + 1669.36 e^{-\sqrt{0.0048}t} = 0$$

$$e^{\sqrt{0.0048}t} (-469.364 e^{\sqrt{0.0048}t} + 1669.36 e^{-\sqrt{0.0048}t}) = e^{\sqrt{0.0048}t} (0)$$

$$-469.364 e^{2\sqrt{0.0048}t} + 1669.36 = 0$$

$$e^{2\sqrt{0.0048}t} = \frac{-1669.36}{-469.364}$$

$$t = \frac{\ln\left(\frac{-1669.36}{-469.364}\right)}{2\sqrt{0.0048}}$$

$$t = 9.1569 \text{ Months}$$

Once the length of the war has been determined, it is entered into $x(t)$ to calculate how many of Milky Way's ships remain after achieving victory.

$$x(9.1569) = 812 e^{\sqrt{0.0048} \times (9.1569)} + 2887.99 e^{-\sqrt{0.0048} \times (9.1569)}$$

$$x(9.1569) = 3062.72 \text{ ships}$$

CONCLUSION

It takes approximately 9 months to annihilate Andromeda's forces. The Milky Way Galactic Navy loses 17% of their fleet, ending the conflict with 3,062 ships intact. Milky Way awards SRJC Math Club for their heroic mathematics and resumes their search for renewable space energy.